

## Investigations of Site Response of Soil Column by True Nonlinear and Linear Method of Analysis

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### ABSTRACT

In this paper, site response of a two layered soil deposit with the assumption of linear and rigid base was analyzed by using linear and true non-linear approaches. Physically, the problem is to predict soil response at selected locations of the profile, such as at the free surface or foundation depth, resulting from the prescribed seismic excitation. The amplification spectrum of the soil column is computed between the top and the bottom of this soil deposit. The true nonlinear analysis will be compared with the linear method of analysis. SHAKE modeling was conducted to compare with the true-non-linear method. Steps involved in ground response analyses to develop site-specific response spectra at a soil site was briefly summarized and illustrated by an example.

**Keywords:** Site Response, Nonlinear, Seismic Excitation, Response Spectra

### INTRODUCTION

Site response analysis is usually the first step of any seismic soil-structure study. Equivalent-linear model is one of the most widely used approaches to model soil nonlinearity. SHAKE (Schnabel et al. 1972). It has been used widely, and name is now used as if it were a common noun. This code based on the multiple reflection theory, and nonlinearity of soil is considered by the equivalent linear method. Unlike the name of "equivalent", this is an approximate method.

Shake uses a frequency domain approach to solve the ground response problem. In simple terms, the input motion is represented as the sum of a series

of sine waves of different amplitudes, frequencies, and phase angles. A relatively simple solution for the response of the soil profile to sine waves of different frequencies (in the form of a transfer function) is used to obtain the response of the soil deposit to each of the input sine waves. The overall response is obtained by summing the individual responses to each of the input sine waves. This section describes the basic mathematics of the process for a problem involving a single soil layer, illustrates how that problem can be solved using a widely available mathematical programming language, and extends the approach to layered systems. To illustrate the basic approach used in SHAKE, consider a uniform soil layer lying on an elastic layer of rock that extends to infinite depth, as illustrated in figure 1. If the subscripts  $s$  and  $r$  refer to soil and rock, respectively, the horizontal displacements due to vertically propagating harmonic s-waves in each material can be written as:

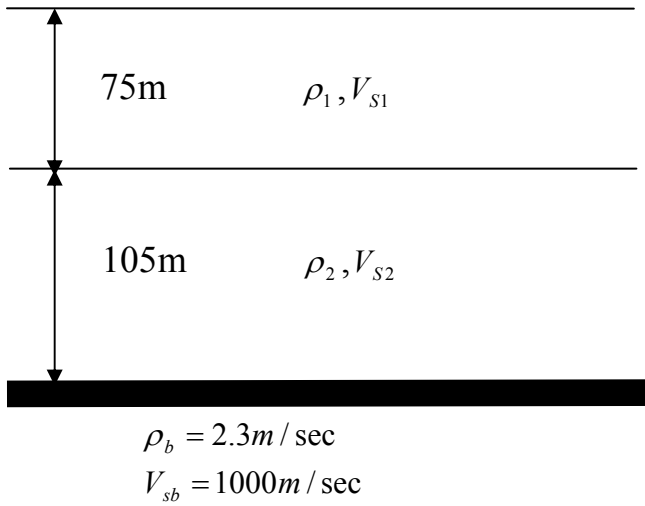
$$u_s(z_s, t) = A_s e^{i(\omega t + k_s^* z_s)} + B_s e^{i(\omega t - k_s^* z_s)} \quad (1)$$

$$u_r(z_r, t) = A_r e^{i(\omega t + k_r^* z_r)} + B_r e^{i(\omega t - k_r^* z_r)} \quad (2)$$

where  $\omega$  is the circular frequency of the harmonic wave and  $k^*$  is the complex wave number. No shear stress can exist at the ground surface ( $z_s=0$ ), so

$$\tau(0, t) = G_s^* \gamma(0, t) = G_s^* \frac{\partial u_s(0, t)}{\partial z_s} = 0 \quad (3)$$

where  $G_s^* = G(1 + 2i\xi)$  is the complex shear modulus of the soil.



**Figure 1. Bedrock half-space interface**

Finn et al. (1978) compared dynamic response of a model ground by three computer codes SHAKE, DESRA and CHARSOIL. DESRA uses hyperbolic model, and CHARSOIL uses Ramberg-Osgood mode. Results by two nonlinear analyses by using DESRA and CHARSOIL are almost the same but SHAKE gives larger shear stress and exhibits larger shear strain than specified. Finn et al. (1978) explained that large amplification comes from the resonance because equivalent linear analysis is a linear analysis. Yoshida and Iai (1998) showed that equivalent linear analysis shows larger peak acceleration because the method calculates acceleration in high frequency range large.

### Solution Methods for Site Response Analysis of Multiple Layers

#### a) Equivalent Linear Method

Because the transfer function is defined as the ratio of the soil surface amplitude to the rock outcrop amplitude, the soil surface amplitude can be obtained as the product of the rock outcrop amplitude and the transfer function. Therefore, the response of the soil layer to a periodic input motion can be obtained by the following steps:

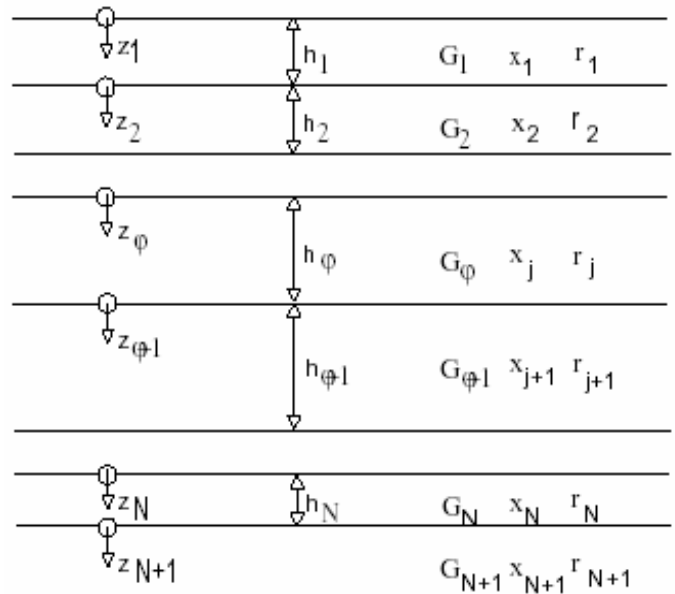
- Express the input (rock outcrop) motion in the frequency domain as a Fourier series (as the sum of a series of sine waves of different

amplitudes, frequencies, and phase angles). For an earthquake motion, this Fourier series will have both real and imaginary parts.

- Define the transfer function. The transfer function will have both real and imaginary parts.
- Compute the Fourier series of the output (ground surface) motion as the product of the Fourier series of the input (bedrock) motion and the transfer function. This Fourier series will also have both real and imaginary parts.
- Express the output motion in the time domain by means of an inverse Fourier transform.

Consider the soil deposit shown as shown in figure 2. Within a given layer, (layer  $j$ ) for the two motions A and B the horizontal displacements will be given by:

$$u_j(z_j, t) = (A_j e^{ik_j^* z_j} + B_j e^{-ik_j^* z_j}) e^{i\omega t} \quad (4)$$



**Figure 2 Equivalent Linear Method Soil Properties**

At the boundary between layer  $j$  and layer  $j+1$ , compatibility of displacements requires that

$$A_{j+1} + B_{j+1} = A_j e^{ik_j^* h_j} + B_j e^{-ik_j^* h_j} \quad (5)$$

Continuity of shear stresses requires that

$$A_{j+1} - B_{j+1} = \frac{G_j^* k_j^*}{G_{j+1}^* k_{j+1}^*} \left( A_j e^{ik_s^* h_j} - B_j e^{-ik_s^* h_j} \right) \quad (6)$$

The effective shear strain of equivalent linear analysis is defined as in Shake calculation:

$$\gamma_{eff} = R_\gamma \gamma_{max} \quad (7)$$

where  $R_\gamma$  is a strain reduction factor often taken as

$$R_\gamma = \frac{M-1}{10} \quad (8)$$

in which  $M$  is the magnitude of earthquake.

While the equivalent linear approach allows the most important effects of nonlinear, inelastic soil behavior to be approximated, it must be emphasized that it remains a linear method of analysis. It is based on the continuous solution of the wave equation, adapted to use with transitory movements by means of the Fast Fourier Transform algorithm. The strain-compatible shear modulus and damping ratio remain constant throughout the duration of an earthquake - when the strains induced in the soil are small and when they are large. Permanent strains cannot be computed and pore water pressures cannot be computed. However, the equivalent linear approach has been shown to provide reasonable estimates of soil response under many conditions of practical importance.

Maximum shear modulus of a layer can be calculated via:

$$v_s = \sqrt{G/\rho} = \sqrt{\frac{Gg}{\gamma}} \quad (9)$$

in which  $G_{max}$  is maximum shear modulus,  $\rho$  is density of the soil,  $\gamma$  is unit weight, and  $g$  is the acceleration of gravity.

#### b) Nonlinear Site Response Analysis

Both time domain and the frequency-domain analyses are used to account for the non-linear effects in site-response problems. The true non-linear and equivalent-linear methods are utilized respectively in the time and frequency domain for the one-dimensional analyses of shear wave

propagation in layered soil media. In the project we used true-nonlinear methods to solve the problem.

#### True-Nonlinear Methods

The soil medium is divided into 5 sub-layers with absolute displacements  $u_j$ , defined at the  $j$  th sub layer, interface and with shear stress,  $\tau_j$ , defined at the mid-points of each interface.

$$\text{Top Layer :} \quad \Delta \ddot{u}_1 = (1/m_1) * \Delta \tau_1$$

$$j \text{ th layer:} \quad \Delta \ddot{u}_j = (1/m_j) * \Delta \tau_j - \Delta \tau_{j-1}$$

$$\text{Half-Space(Interface):} \Delta \ddot{u}_j = (1/(m_b + (1/2 \Delta t \rho_b v_{sb}))) * \Delta \tau_j - \Delta \tau_{j-1}$$

$$m_b = 1/2 \rho_n \Delta x_n \quad ; \quad \Delta \rho_b = \rho_b v_{sb} [2 \Delta \dot{u}_1 - \Delta \dot{u}_b]$$

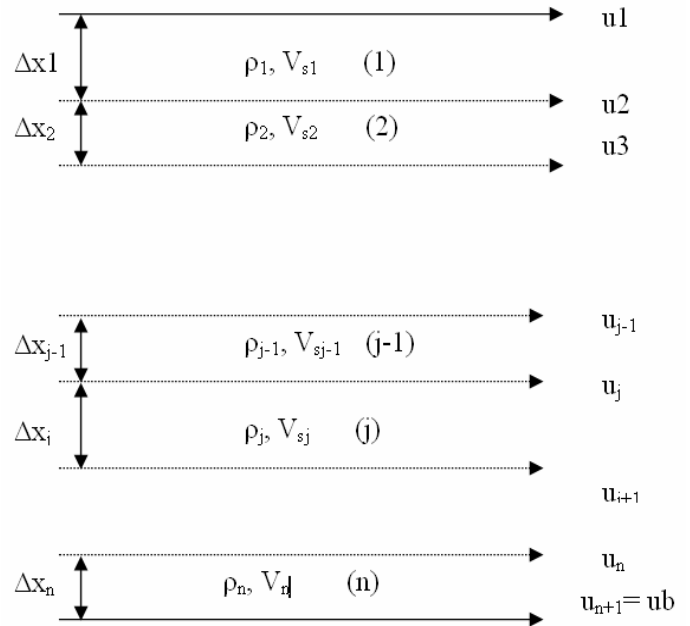
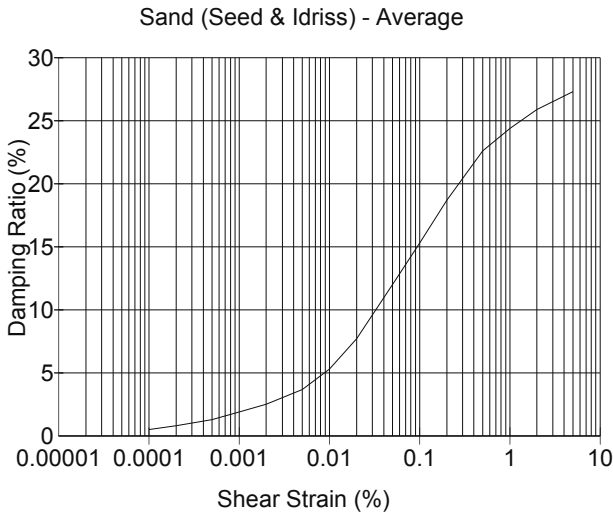


Figure 3 Nonlinear Method Soil properties definition

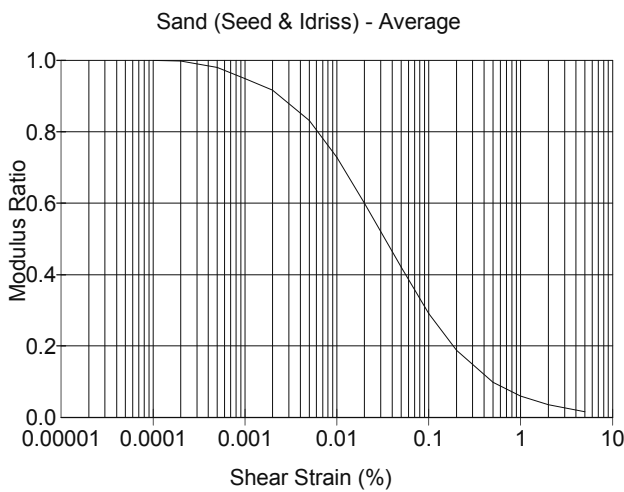
#### Numerical Results

The results of site response analyses were presented in terms of acceleration time history and response spectra. SHAKE91 (Idriss 1992) uses soil properties linear equivalent with an iterative procedure to obtain properties compatible with the deformations developed in each stratum. The input and output motion of the soil medium is given from Figure 4 to Figure 9. The comparison of linear elastic numerical analysis by using SHAKE91 and True nonlinear analysis are given in figure 10 to

figure 12, and the results of are summarized in table1.



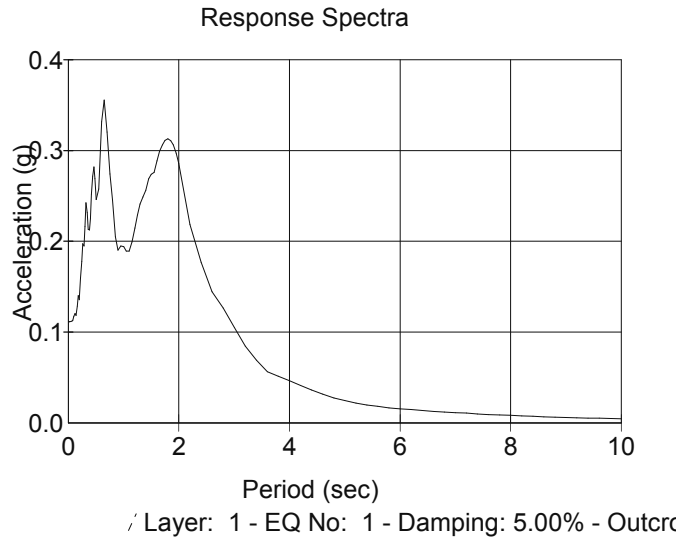
**Figure 4 Input Motions: Shear Strain Damping Ratio**



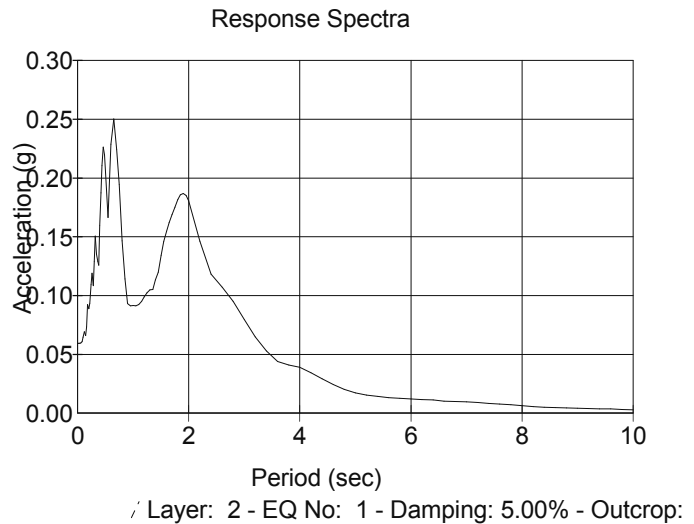
**Figure 5 Input Motions: Shear Strain Shear Modulus**

Total thickness of the soil deposits is 180m. The top soil is silty sand (SM) with 75 m thickness and  $19.5 \text{ kN/m}^3$  mass density ( $\rho_1$ ) and 420m/sec shear wave velocity  $V_{s1} = 420 \text{ m/sec}$ . Lower layer is silty gravel (GM) with mass density,  $\rho_2 = 1.95 \text{ t/m}^3$  and the shear wave velocity and  $V_{s2} = 600 \text{ m/sec}$  105 m thickness. The shear wave velocity of the half-space interface is 1000m/s. The solution algorithm used in SHAKE assumes viscous soil damping which represents using a complex shear modulus. Viscous damping implies behavior that would be characterized by elliptical stress-strain loops. Because actual stress-strain loops are seldom

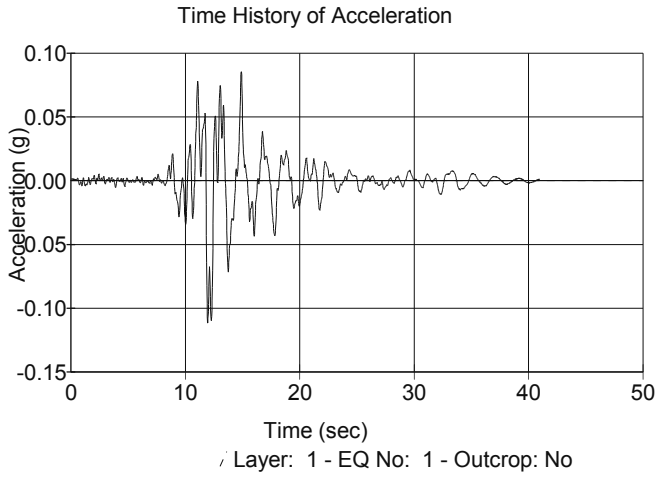
elliptical, an equivalent damping ratio is used – the equivalent damping ratio is equal to the damping ratio that would be computed based on the area within the hysteresis loop, the secant shear modulus, and the maximum shear strain. The relationship between this equivalent damping ratio and shear strain is characterized by means of a damping curve. 5% damping ratio is used in this study.



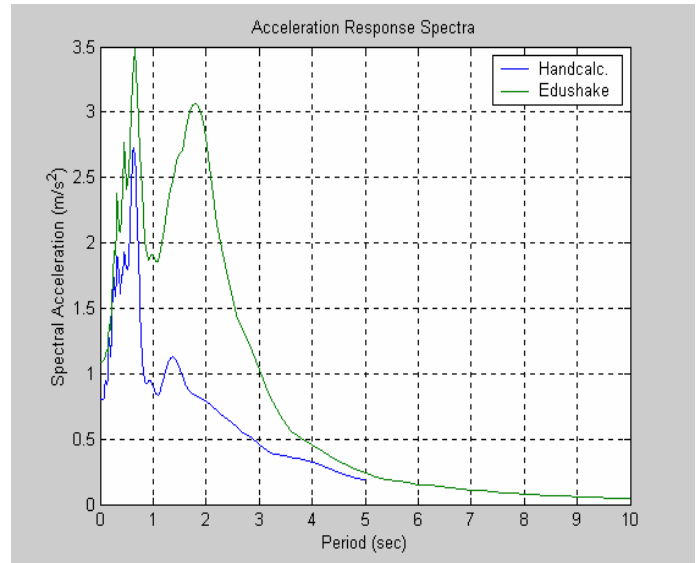
**Figure 6 Output Motions Top Layer Response Spectra**



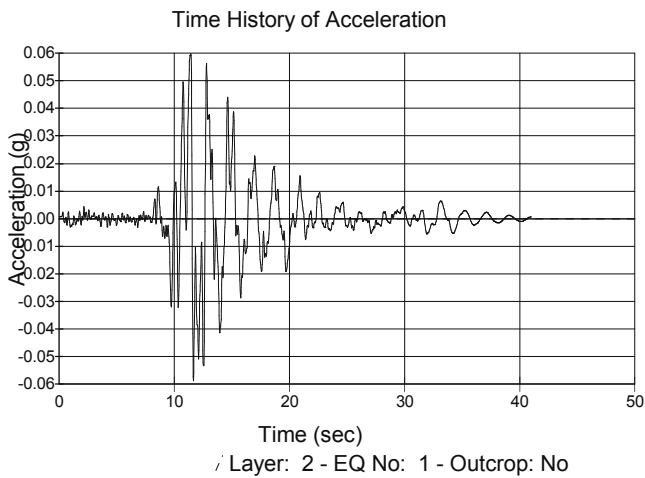
**Figure 7 Output Motions Second Layer Response Spectra**



**Figure 8 Top Layer Time History of Acceleration**



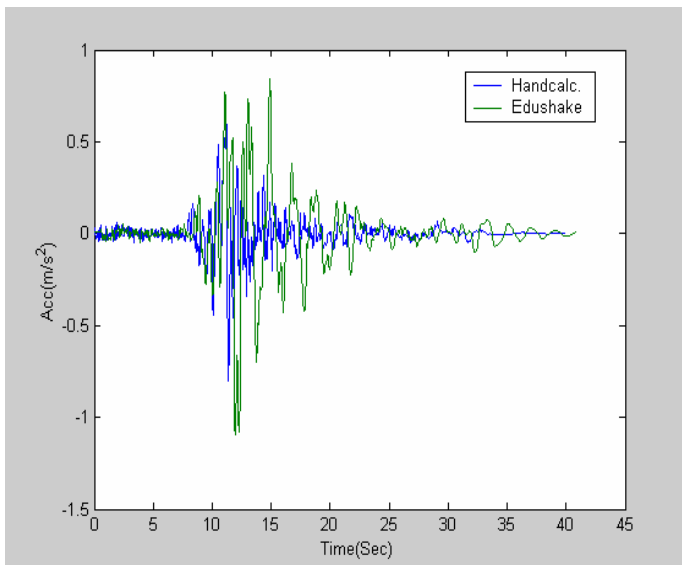
**Figure 11 Comparisons of Acceleration Response Spectra for the Top Layer**



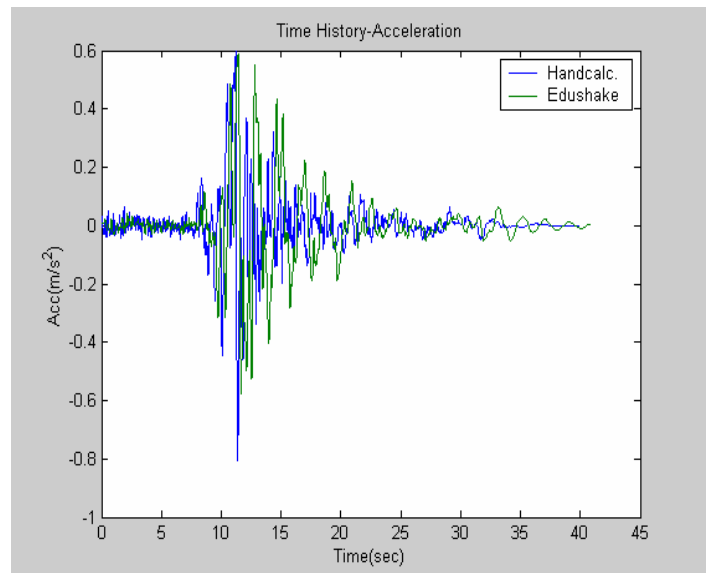
**Figure 9 Second Layer Time History of Acceleration**

**Table 1 Summary of the Site Response Analysis**

	Max. Acceleration (m/s <sup>2</sup> )	
	Mat-lab	Shake
First Layer	<b>0.61</b>	<b>0.82</b>
Second Layer	<b>0.8</b>	<b>1.06</b>
Spectral acceleration(m/s <sup>2</sup> )	<b>2.7</b>	<b>3.5</b>



**Figure 10 Comparison of Acceleration & Time History Relation for the Top Layer**



**Figure 12 Comparison of Acceleration & Time History Relation for the Second Layer**

## Summary and Conclusions

A parametric study was conducted to assess the effect of numerical analysis method on site response. In order to study the linear and nonlinear behavior of soil column under dynamic loading, the ground motions of two layer soil column was analyzed in the paper. The dynamic analysis was carried out by means of the program SHAKE91 and by implementing nonlinear site response method into Mat-Lab. It has been demonstrated from maximum acceleration distribution along depth and also from spectrum ratios that equivalent linear analysis calculates larger peak acceleration. Because linear site response analysis calculates acceleration in high frequency range, the method gives higher acceleration. Based on one-dimensional site response analyses, the effect of nonlinear soil behavior is one of the key factors for response spectra.

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